Dipole? The Q^2 dependece of the axial nucleon form factor extracted from CCQE MiniBooNE data

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Motivation

($\star$) One the main systematic errors in the neutrino oscillation experiments are those associated with (anti)neutrino cross sections.

($\star$) In these experiments neutrino beams are peaked in the $0.3 - 5 GeV$ energy range, where CCQE and resonance production process are dominant.

($\star$) CCQE interaction represents a two-particle scattering process with lepton and nucleon in the final state. Neutrino energy can be evaluated based on kinematics of the outgoing final state particles.

($\star$) CCQE events selection criteria is strongly influenced by ν-beam, target material, and detector technology. Therefore various selection techniques are applied in this experiments (tracking & calorimeter, Cerenkov detector).

($\star$) Various the modern neutrino experiments measured CCQE $d\sigma/dE_\mu d\cos\theta$, $d\sigma/dQ^2$ inclusive and total cross sections.

Motivation

- $(\star)$ With the assumptions of CVC and PCAC the only undetermined form factor is the axial form factor $F_A(Q^2)$

- $(\star)$ The analysis are mainly based on the Fermi gas model (RFGM) with the dipole parameterization of F_A with free parameter of the axial mass M_A.
  $$F_A(Q^2) = F_A(0)/(1 + Q^2/M_A^2)^2$$

- $(\star)$ Formal averaging of M_A values, *i.e.* average value of axial mass
  $$M_A \approx 1.026 \pm 0.021 GeV$$

- $(\star)$ Results of the modern experiments K2K, (Sci)MiniBooNE, NOMAD, MINERvA, T2K
  $$M_A \approx 1.0 \div 1.35 GeV$$

- $(\star)$ Absolute values of the differential and total cross sections measured by MiniBooNE are about 30% larger as compared to the NOMAD result.

Motivation

Several studies have tried to address these discrepancies.

- $(\star)$ Modified nuclear models which are based on Impulse Approximation (IA) have been used to find an axial mass close to MiniBooNE result $M_A \approx 1.3 \div 1.4 GeV$

- $(\star)$ Other models include effects of multinucleon excitation such as the meson-exchange currents (MEC) and isobar current (IC). These contributions has been found sizable ($\approx 30\%$) and allow one to reproduce the MiniBooNE cross sections with $M_A \sim 1.03 GeV$.

- $(\star)$ The transverse enhancement (TE) effective model to account for MEC also allows one to describe the MiniBooNE total cross section with $M_A \approx 1.014 GeV$.

- $(\star)$ Fully relativistic calculations beyond IA are extremely difficult and future work is need to improve this approach.

- $(\star)$ The assumption of the dipole ansatz is a crucial element in many of these studies.

($\star$) The value of $F_A(Q^2)$ as a function of Q^2 were extracted from pion electroproduction data on free proton (V.Bernard et al. 2002) and from $d\sigma/dQ^2$ cross section data for neutrino scattering on deuterium (A. Bodek et al. 2007).

($\star$) A reasonable description of $F_A(Q^2)$ by dipole approximation with $M_A \approx 1 GeV$ was found.

($\star$) We extended the procedure developed (H.Budd et al. 2005) for extraction of $F_A(Q^2)$ from deuterium data to heavier nuclei and extracted in the RFGM, RFGM+TE, and RDWIA approaches Q^2-dependence of axial form factor from the MiniBooNE measured $d\sigma/dQ^2$ cross sections.

Formalism of the CCQE Scattering

Formalism and Model

In the **inclusive reactions**

$$\nu(k_\nu) + A(p_A) \to \mu'(k_\mu) + X$$

only the outgoing lepton is detected and the differential cross sections can be written as

$$\frac{d^3\sigma^{cc}}{d\varepsilon_\mu d\Omega_\mu} = \frac{1}{(2\pi)^2} \frac{|\mathbf{k}_\mu|}{\varepsilon_\nu} \frac{G^2 \cos^2 \theta_c}{2} L_{\mu\nu}^{(cc)} W^{\mu\nu(cc)}$$

where $L_{\mu\nu}$ is CC lepton tensor and $W^{\mu\nu}$ is weak CC hadron tensor.

- (●) All the nuclear structure information and FSI effects are contained in $W_{\mu\nu}$, which is given by a bilinear product of the transition matrix elements of the nuclear CC operator J_μ between the initial nucleus state $|A\rangle$ and the final state $|X\rangle$ as

$$W_{\mu\nu} = \sum \langle X|J_\mu|A\rangle\langle A|J_\nu^\dagger|X\rangle = \langle J_\mu \cdot J_\nu^\dagger\rangle$$

  where the sum is taken over undetected states.

- (●) In Impulse Approximation (IA) the incoming neutrino interacts with only one nucleon. The nuclear current is written as the sum of single-nucleon currents.

Formalism and Model

- The single-nucleon charged current has V-A structure $J^\mu = J_V^\mu + J_A^\mu$ and for a vertex function $\Gamma^\mu = \Gamma_V^\mu + \Gamma_A^\mu$ we use a free nucleon vector current vertex function

$$\Gamma_V^\mu = F_V(Q^2)\gamma^\mu + i\sigma^{\mu\nu}q_\nu F_M(Q^2)/2m,$$

  where $\sigma_{\mu\nu} = i[\gamma^\mu\gamma^\nu]/2$, F_V and F_M are the weak vector form factors.

- We use the MMD approximation [P.Mergell et al (1996)] of the vector nucleon form factors with de Forest prescription (T. de Forest, 1983) and the Coulomb gauge for off-shell extrapolation of Γ_V^μ.

- The axial current vertex function can be written in term of the axial $F_A(Q^2)$ and pseudoscalar $F_p(Q^2)$ form factors

$$\Gamma_A^\mu = F_A(Q^2)\gamma^\mu\gamma_5 + F_P(Q^2)q^\mu\gamma_5.$$

  The pseudoscalar form factor $F_P(Q^2)$ can be written in form

$$F_p(Q^2) = 2mF_A(Q^2)/(m_\pi^2 + Q^2) = F_A(Q^2)F_P'(Q^2),$$

  where $F_P'(Q^2) = 2m/(m_\pi^2 + Q^2)$. Then axial vertex function $\Gamma_A^\mu = F_A(Q^2)[\gamma^\mu\gamma_5 + F_P'(Q^2)q^\mu\gamma_5]$ and F_A is undetermined form factor.

Model

- In independent particle shell model IPSM the relativistic bound-state functions Φ are solutions of a Dirac equation, derived within a relativistic mean field approach in the $\sigma - \omega$ model [B.Serot et al. 1986].

- According to JLab data [D. Dutta et al. (2003), J.J. Kelly 2005] the occupancy of the IPSM orbitals of ^{12}C equal 89% on average. We assume that the missing strength 11% can be attributed to the short-range NN-correlations in the ground state of nucleus.

- The contribution of the two-body currents, such as MEC and IC are not considered.

- In the RDWIA the FSI effects are taken into account. The distorted-wave function of the knocked out nucleon is evaluated as a solution of a Schrödinger equation [LEA code J.J. Kelly, 1995] containing a phenomenological relativistic optical potential [E.Cooper et al.,1993].

- In Fermi Gas model for carbon we use p_F=221 MeV/c and ϵ_b=25 MeV. The RFGM does not account for nuclear shell structure, FSI effect, and the presence of NN-correlations.

Method of F_A extraction

Hadron tensor

(●) In IA hadron tensor is given by products of the transition matrix elements of nucleon weak current operator J_μ, i.e.
$$W_{\mu\nu} = \langle J_\mu J_\nu^\dagger \rangle.$$

The angle brackets denote products of matrix elements, appropriately averaged over initial states and summed over final states.

(●) Axial vector current can be factorized in the form
$J_A = F_A(Q^2)J_A'(Q^2)$, where
$$J_A' = \gamma^\mu \gamma_5 + F_p'(Q^2)q^\mu \gamma_5$$

and weak current can be expressed as
$$J = J_V(Q^2) + F_A(Q^2)J_A'(Q^2)$$

(●) The expression for the hadron tensor then given by
$$W_{\mu\nu} = W_{\mu\nu}^V + F_A^2(Q^2)W_{\mu\nu}^A + hF_A(Q^2)W_{\mu\nu}^{VA},$$

where $W_{\mu\nu}^V = \langle (J_V)_\mu (J_V)_\nu^\dagger \rangle$, $W_{\mu\nu}^A = \langle (J_A')_\mu (J_A')_\nu^\dagger \rangle$,

$W_{\mu\nu}^{VA} = \langle (J_V)_\mu (J_A')_\nu^\dagger + (J_A')_\mu (J_V)_\nu^\dagger \rangle$, and h is 1 for ν and -1 for $\bar\nu$.

Cross Section

- Contracting $W_{\mu\nu}$ with lepton tensor $L^{\mu\nu}$ we obtain

$$d\sigma/dQ^2 = \sigma^V + F_A^2(Q^2)\sigma^A + hF_A(Q^2)\sigma^{VA},$$

where $\sigma^V = d\sigma/dQ^2(F_A = 0)$ is the vector cross section due to the vector component of weak current J_V,
$\sigma^A = d\sigma/dQ^2(F_V = F_M = 0, F_A = 1)$ is axial cross section due to J'_A and doesn't depend on the vector form factors.

$$\sigma^{VA} = [\sigma(F_A = 1) - \sigma^V - \sigma^A],$$

arising from interference between the vector and axial currents and
$\sigma(F_A = 1) = d\sigma/dQ^2(F_A = 1)$.

- In the case of ν scattering on free or "quasi free" nucleon the cross sections σ^V, σ^A, and σ^{VA} can be expressed in terms of the vector form factors. For instance

$$\sigma^{VA} = \frac{G_F^2}{2\pi}cos^2\theta_c\frac{Q^2}{m\varepsilon_\nu}(1 - Q^2/4m\varepsilon_\nu)[F_V(Q^2) + F_M(Q^2)],$$

is going to zero at $Q^2 \to 0$ and decreases with ε_ν.
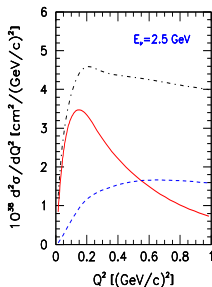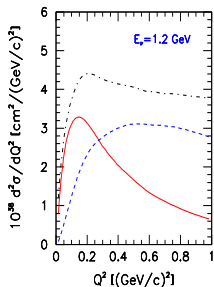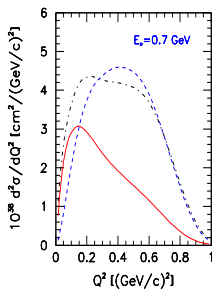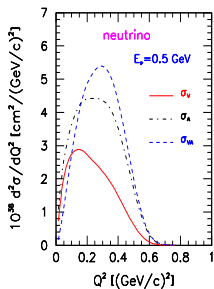
Cross Section

Differential cross sections σ_V, σ_A, and σ_{VA} vs Q^2 for neutrino scattering off carbon calculated in the RDWIA for neutrino energy: $\varepsilon_\nu = 0.5, 0.7, 1.2$ and $2.5 GeV$.

σ_V has maximum at $Q^2 \approx 0.15 (GeV/c)^2$ and depends slowly on ε_ν

σ_A is dominant at $\varepsilon_\nu > 1 GeV$ in the range $Q^2 > 0.2 (GeV/c)^2$ and slowly decreases with Q^2

$\sigma_{VA} \sim 1/\varepsilon_\nu$ and depend slowly on Q^2 at $Q^2 > 0.3 (GeV/c)^2$

Method

- In neutrino experiments $d\sigma/dQ^2$ are measured within wide range of neutrino energy spectrum and flux-integrated cross section

$$\langle\frac{d\sigma^{\nu,\bar{\nu}}}{dQ^2}\rangle = \int_{\varepsilon_{min}}^{\varepsilon_{max}} P_{\nu,\bar{\nu}}(\varepsilon_\nu)\frac{d\sigma^{\nu,\bar{\nu}}}{dQ^2}(Q^2,\varepsilon_\nu)d\varepsilon_\nu$$

  can be extracted from data.

- In terms of flux-integrated $\langle\sigma^V\rangle$, $\langle\sigma^A\rangle$ and $\langle\sigma^{VA}\rangle$ we can write

$$\langle\frac{d\sigma^{\nu,\bar{\nu}}}{dQ^2}\rangle = \langle\sigma^V\rangle^{\nu,\bar{\nu}} + F_A^2\langle\sigma^A\rangle^{\nu,\bar{\nu}} + F_A\langle\sigma^{VA}\rangle^{\nu,\bar{\nu}}$$

  Values of F_A can be extracted as the solution this equation, using data for $\langle d\sigma^{\nu,\bar{\nu}}/dQ^2\rangle$ and $\langle\sigma^i\rangle$ ($i = V, A, VA$) averaged in Q^2-bins $\Delta Q^2 = Q_{j+1}^2 - Q_j^2$

$$\langle\sigma^i\rangle_j = \frac{1}{\Delta Q^2}\int_{Q_j^2}^{Q_{j+1}^2}\langle\sigma^i(Q^2)\rangle dQ^2$$

- In the case of neutrino scattering on deuterium (quasi-free nucleon) this procedure was applied [H.Budd et al.,2005, A.Bodek et al.,2008] for extraction $F_A(Q^2)$ vs Q^2.

RESULTS

MiniBooNE Result

- The MiniBooNE ν_μ ($\bar{\nu}_\mu$) CCQE flux-integrated differential cross section $d\sigma/dQ^2$ were extracted as function of Q^2 in the range $0 < Q^2 < 2(GeV/c)^2$ [A.A. Aguilar-Arevalo et al. 2010, 2013]. The total normalization error on neutrino (antineutrino) cross section measurement is 10.7% (17.2%).

- Within the Fermi Gas model with dipole parametrization of $F_A(Q^2)$ the "shape-only" fit yelds the model parameter $M_A = 1.35 \pm 0.17 GeV$.

- To extract $F_A(Q^2)$ vs Q^2 from the MiniBooNE ν_μ and $\bar{\nu}_\mu$ flux-folded $d\sigma/dQ^2$ cross sections we calculated the flux-integrated $\langle \sigma^V \rangle$, $\langle \sigma^A \rangle$, and $\langle \sigma^{VA} \rangle$ cross sections with Booster Neutrino Beamline ν_μ and $\bar{\nu}_\mu$ fluxes [A.A. Aguilar-Arevalo et al., 2009]. The measured cross sections with "shape-only" error were used in equation.

Flux-integrated cross sections
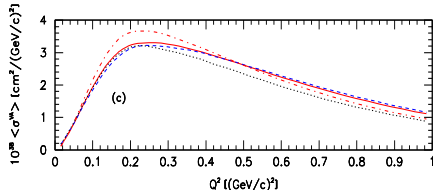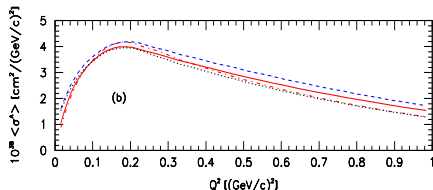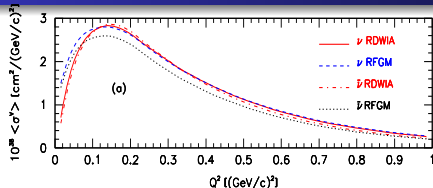
Flux-integrated $\langle\sigma^V\rangle^{\nu,\bar\nu}$, $\langle\sigma^A\rangle^{\nu,\bar\nu}$, and $\langle\sigma^{VA}\rangle^{\nu,\bar\nu}$ cross sectionas as functions of Q^2. The cross section were calculated in the RDWIA and RFGM approaches. In the region $Q^2 < 0.25 (GeV/c)^2$ the Fermi gaz model results are higher ($\approx 10\%$) than those obtained within RDWIA.

Neutrino cross section and form factor

Flux-integrated $\langle d\sigma/dQ^2\rangle^\nu$ cross section per neutron target for the neutrino CCQE scattering (upper panel) and the normalized axial form factor $F_A(Q^2)/F_A(0)$ extracted from the MiniBooNE data (lower panel). Upper panel: calculations from the RDWIA with $M_A = 1.37 GeV$ and RFGM with $M_A = 1.36 GeV$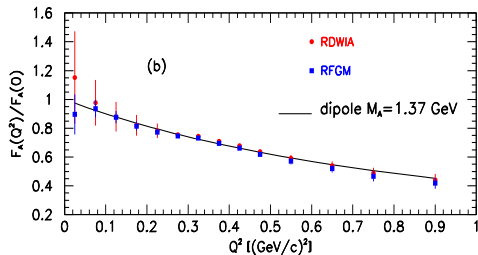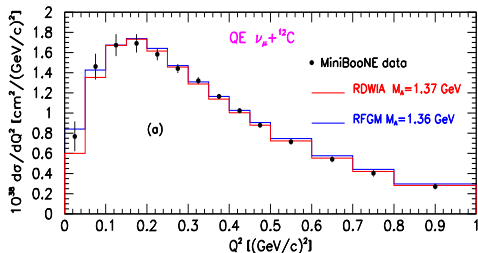. Lower panel: filled circles (filled squares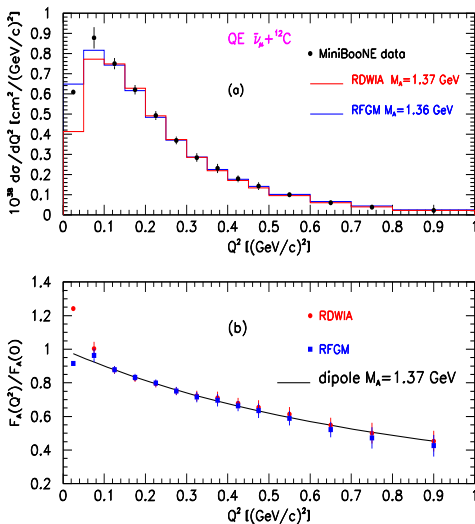) are the axial form factor extracted within the RDWIA (RFGM) and solid line is the result of the dipole parametrization with $M_A = 1.37 GeV$.

# Antineutrino cross section and form factor

Flux-integrated $\langle d\sigma/dQ^2\rangle^\nu$ cross section per proton target for the antineutrino CCQE scattering (upper panel) and the normalized axial form factor extracted from the MiniBooNE data (lower panel). There is an agreement between the calculated and measured cross sections. The value of the axial form factors extracted within the RDWIA are similar to the RFGM result. A good match between the dipole parametrization with $M_A = 1.37\,GeV$ and extracted form factors is observed.

Difference of the cross sections and form factor
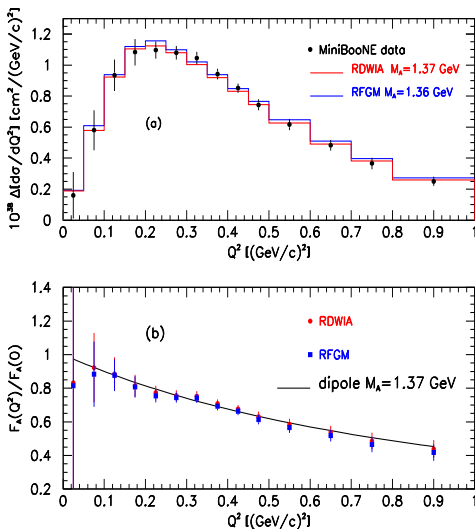
The difference of the measured cross sections $\delta[d\sigma/dQ^2] = \langle d\sigma/dQ^2\rangle^\nu - \langle d\sigma/dQ^2\rangle^{\bar\nu}$ (upper panel) vs Q^2 compared with the RDWIA and RFGM calculations. Also shown are the normalized axial form factor extracted in this approaches from $\Delta[d\sigma/dQ^2]$ (lower panel). A good match between the calculated and measured differences $\delta[d\sigma/dQ^2]$ is observed. The extracted axial form factors also agrees well with the dipole approximation with $M_A = 1.37 GeV$.

- To explane the discrepancy between the low (MiniBooNE) and high (NOMAD) energy (anti)neutrino CCQE cross section and M_A measurement it was assumed [A. Bodek et al., 2011] that enhancement in the transverse (anti)neutrino CCQE cross section are modifies $F_M(Q^2)$, for bound nucleon at low $Q^2 \approx 0.3(GeV/c)^2$.

- If the transverse enhancement (TE) orginates from MEC, inhanement in the longitudinal and axial contribution is small. Therefore, in this model $F_V(Q^2)$ and $F_A(Q^2)$ are the same as for free nucleon. A TE function was proposed for carbon target.

- To study the TE effects on the extracted $F_A(Q^2)$ we compare results of the RFGM ($M_A = 1.36 GeV$) and FRGM+TE ($M_A = 1.014 GeV$) [J. T. Sobczyk, 2012] with the TE function from [A. Bodek, 2011].
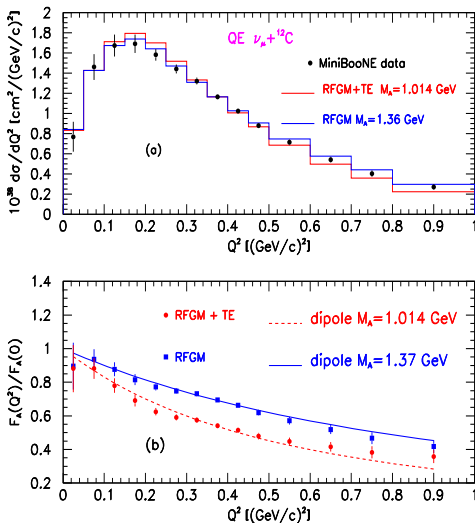
In the range $Q^2 < 0.5(GeV/c)^2$ the cross sections calculated within the RFGM+TE model is in agreement with data and RFGM prediction. At $Q^2 > 0.7(GeV/c)^2$ the RFGM+TE with $M_A = 1.014 GeV$ is lower than measured cross section.

Neutrino cross sections with TE
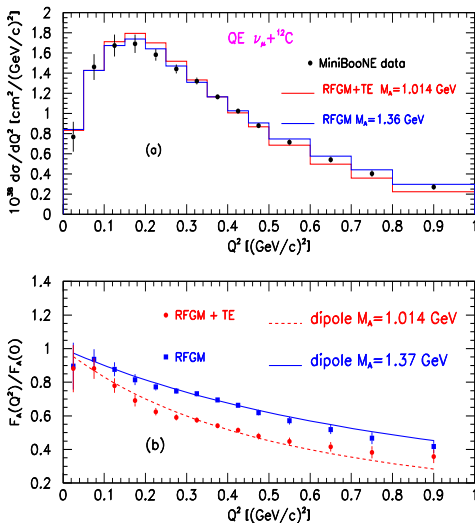
The shape of the Q^2-dependence of the axial form factor, extracted within RFGM+TE approach can't be well described by the dipole ansatz. In the region $Q^2 \approx 0.2 - 0.3 (GeV/c)^2$, where enhancement in the $F_M(Q^2)$ was assumed, the extracted values of F_A are lower than those predicted from dipole approximation.

Neutrino cross sections with TE
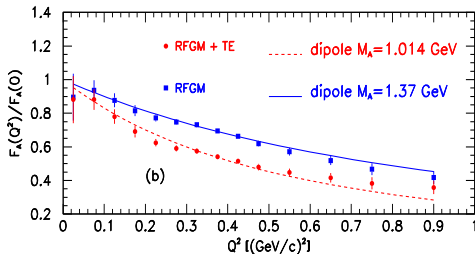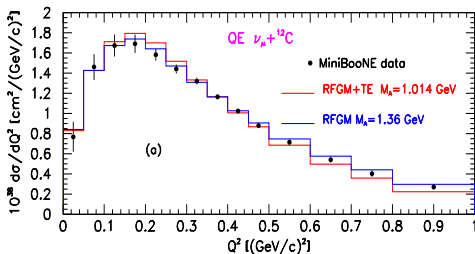
At $Q^2 > 0.6(GeV/c)^2$ the enhancement function disappears with higher values of Q^2. And the values of F_A extracted in RFGM+TE model start approaching to those extracted in the RFGM.

For self-consistent description of the MiniBooNE data when using transverse responce function with the enhancement one must use some non-dipole parametrization for $F_A(Q^2)$.

- The dipole approximation of the nucleon form factors assumes that nucleons have a simple exponential spatial charge distribution $\rho(R) = R_0 \exp(-R/R_0)$, where R_0 is the scale of nucleon radius.
- Form factors are related in the non-relativistic limit to the Fourier transform of the charge, magnetic momentum, and axial charge distribution. Above $\rho(R)$ distribution yields the dipole form $F_D^{V,M,A} = F_0^{V,M,A}/(1 + Q^2/M_{V,A}^2)^2$
  The magnetic momentum distribution were assumed to have the same spatial dependence as the charge distribution.
- From the analysis of the MiniBooNE measured CCQE cross sections follows that Q^2-dependence of the axial and vector form factor are correlated. If one uses the dipole approximation for vector form factors one must use the dipole ansatz and for the axial form factor.
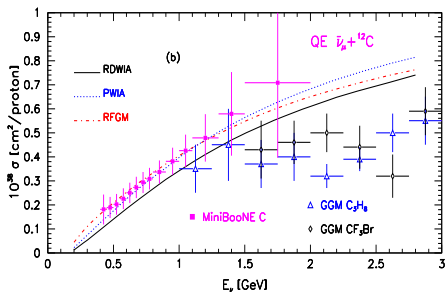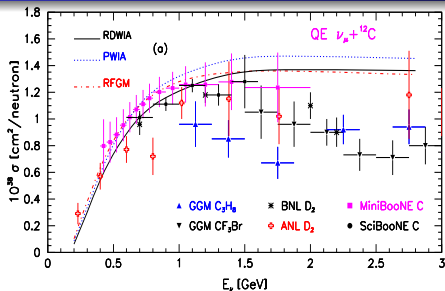
Total cross section for CCQE scattering of muon neutrino (per neutron, upper panel) and antineutrino (per proton, lower panel) on carbon as a function of incoming (anti)neutrino energy. The solid line is the RDWIA results ($M_A = 1.37 GeV$), dashed-dotted line is the RFGM result ($M_A = 1.36 GeV$) and dotted line is the PWIA ($M_A = 1.37 GeV$) result.

CONCLUSIONS

Conclusions

- Method for extraction $F_A(Q^2)$ from the measured flux-integrated $d\sigma/dQ^2$ cross section of CCQE (anti)neutrino scattering off nuclei is presented.
  - ⋆ It is based on the fact that $d\sigma/dQ^2$ cross section can be written as sum of the vector σ^V, axial σ^A, and vector-axial σ^{VA} cross sections.
- In our analysis we used the cross sections with "shape-only" error measured in the MiniBooNE experiment.
  - ⋆ In the RDWIA, RFGM, and RFGM+TE approaches we calculated the flux-integrated $\langle\sigma^V\rangle$, $\langle\sigma^A\rangle$, and $\langle\sigma^{VA}\rangle$ cross sections, which were used for extraction of $F_A(Q^2)$ vs Q^2
  - ⋆ The value of $F_A(Q^2)$ extracted in the RDWIA and Fermi gas model agree well with the dipole approximation result with $M_A = 1.37 GeV$. Whereas we found that there is a disagreement between the from factors extracted in the RFGM+TE approach and predicted by dipole approximation with $M_A = 1.014 GeV$.
- We conclude that Q^2-dependence of the axial and vector form factors are correlated.